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'CRUDE' MULTI-FACTOR PRODUCTIVITY (MFP)

Methodological note[1]

Introduction

In addition to the capital productivity indicators (CAPI) and labour productivity indicators (LPI), a measure of 'crude' multi-factor productivity growth (MFP)[2] is provided **as experimental statistics**. The measure is referred to as 'crude' as it is based on growth rates of capital stocks (in real terms) and hours worked and not on capital and labour services as input measures in more sophisticated measures of MFP. This is the reason why this indicator is experimental. The indicator is directly based on national accounts labour input and total capital stocks data, without need for assumptions or additional information (to derive quality adjusted labour input or capital services) which are used for more sophisticated multi-factor productivity measures.

This measure and the contributions to growth of value added are provided at the total economy level and on a yearly basis.

Compared to labour productivity growth, which represents a higher level of output for one additional hour worked (and analogously capital productivity), MFP growth accounts for the overall efficiency with which labour and capital are combined in a production process. In simple terms, growth in MFP can be described as the change in the volume of output (expressed in terms of real gross value added) that cannot be explained by changes in the quantity and quality of capital and labour inputs used to generate that output. The MFP measure is indeed calculated as residual between the real growth of value added and the weighted growths of volume of inputs (labour and capital). Therefore, MFP includes all factors which are not taken into account on the input side of value-added generation (e.g. influence of weather conditions, etc.). Particularly, included in MFP is the consequences of economic cycles as the total stock of capital is taken into consideration and not only the part of this total stock which is effectively used in production (given by the degree of utilisation).

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[1] Written in cooperation with The Vienna Institute for International Economic Studies (wiiw) within Eurostat's 'Growth and Productivity Accounts' project. The methodological note benefitted from work of the Task Force on productivity indicators (2019-2021).

[2] See Timmer, M.P., R. Inklaar, M. O'Mahony and B. van Ark (2010), Economic Growth in Europe. A Comparative Industry Perspective, Cambridge University Press.

In other terms, if labour and capital inputs remained unchanged between two periods, any changes in output would reflect changes in MFP. Conversely, if labour and capital inputs are growing at the same rate as output, MFP growth would be zero.

Compared to other calculations of MFP growth (e.g. by National Statistical Institutes, OECD[3], or EU KLEMS[4]), the 'crude' MFP does not rely on 'capital services' and 'labour services' growth but on the growth of capital stocks and total hours worked taken from national accounts. As such, the 'crude' MFP does not include changes in labour composition (together with hours worked growth resulting in 'labour services' growth) or valuation of the different capital asset types or change in the quality of assets composing the capital stock (i.e. capital services). These more sophisticated measures (based on the growth of capital and labour services) require various assumptions and more data-intensive calculations.

Therefore, 'crude' MFP growth includes the changes in quality that are not taken into account by labour input (hours worked) and by capital input (capital stock). Thus, the difference between labour services growth and the growth of hours worked (i.e. the labour composition effect) is still included in the 'crude' MFP growth. Similarly to changes in labour composition, changes in capital composition (i.e. growth rates of asset types weighted by their user costs) are part of the 'crude' MFP growth.

Overview of indicators provided

Table 1 provides an overview of published indicators. These are based on growth rates of gross value added, net capital stocks and hours worked (in logarithmic terms). The 'crude' multi-factor productivity indicator is provided for the total economy, for any EU Member State, in terms of index with reference year 2015. The contributions to growth of value added (in percentage points) are provided for labour input (expressed in terms of hours worked), capital (expressed in terms of net capital stock in chain linked volume at replacement cost - 2015) and for multifactor productivity and in this case it can be also interpreted as MFP growth rate[5]. These three components sum up to value added growth and ease analyses on productivity.

[3] Organisation for Economic Co-operation and Development.

[4] EU KLEMS is an EU project that has originally been funded by the European Commission, Research Directorate General as part of the 6th Framework Programme, Priority 8, 'Policy Support and Anticipating Scientific and Technological Needs'. KLEMS refers to capital (K) and labour (L) inputs and when considering gross output production to energy (E), material (M), and services (S).

[5] For further details, refer to the OECD methodological note on productivity statistics.

Table 1 – Overview of provided indicators

Indicator	Description	Unit	Breakdown	Frequency
'Crude' MFP growth	'Crude' multi-factor productivity growth (NA-based approach)	Index, 2015=100	Total economy	Annual data from 2000 onwards
Growth contributions	Contributions to growth of value added of labour input and capital input	Percentage points of value added growth	Total economy	Annual data from 2000 onwards

Note: 'Crude' MFP growth and contributions to growth are calculated in logarithmic terms of gross value added, net capital stocks and hours worked.

Definition and calculation

To calculate 'crude' MFP growth, real value added, total capital stock in chain-linked volumes, hours worked and compensation of employees (in nominal terms) are needed. According to the literature[6], growth rates of MFP are calculated as log-differences.

The growth accounting equation[7] to calculate 'crude' MFP growth (based on value added) is given by:

$$\Delta \ln \mathrm{MFP_t} \equiv \Delta \ln \mathrm{V_t} - \bar{v}_{\mathrm{K},t} \Delta \ln \mathrm{K_t} - \bar{v}_{\mathrm{L},t} \Delta \ln \mathrm{H_t} \qquad (1)$$

Here, leaving out time subscript (t), $\Delta \ln \mathrm{V}$ is the log growth rate of value added (in chain-linked volumes reference price 2015), $\Delta \ln \mathrm{K}$ denotes the log-growth rate of total fixed assets (N11N) in chain-linked volumes reference price 2015, and $\Delta \ln \mathrm{H}$ denotes the log-growth rate of hours worked. The variables $\bar{v}_{\mathrm{K}}$ and $\bar{v}_{\mathrm{L}}$ denote the Divisia shares[8] of capital and labour costs, calculated as the average of the shares in value added calculated in two subsequent periods (years) fulfilling $\bar{v}_{\mathrm{K}} + \bar{v}_{\mathrm{L}} = 1$[9]. The 'crude' MFP growth rate (based on value added) is calculated as a residual and is denoted by $\Delta \ln \mathrm{MFP}$.

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6 This follows Jorgenson, D.W., M.S. Ho and K.J. Stiroh (2005), Productivity. Information Technology and the American Growth Resurgence, MIT Press, and Timmer, M.P., R. Inklaar, M. O'Mahony and B. van Ark (2010), Economic Growth in Europe. A Comparative Industry Perspective, Cambridge University Press.

7 Based on a Cobb-Douglas production function.

8 These are defined as $\bar{v}_{f,t} = (v_{f,t-1} + v_{f,t})/2$ and have first been introduced by François Divisia in 1926; see Diewert, W.E. 1993. The early history of price index research. Chapter 2 of Essays in Index Number Theory, Volume I, W.E. Diewert and A.O. Nakamura, editors. Elsevier Science Publishers, B.V. These are used to weigh the growth rates over the two periods of change.

9 This implicitly assumes that there are constant returns to scale, mark-ups are zero and perfect competition in product and factor markets and the rates of returns are derived endogenously. (see e.g. Measuring Capital, OECD Manual, 2009, pages 68-70 for details).

These calculations require the shares of labour and capital income in value added in nominal terms, i.e. $\bar{v}_K$ and $\bar{v}_L$ respectively. To do so, labour income should be estimated first as it is not directly provided by national accounts as such. Labour income (LAB) is estimated as follows:

$$\mathrm{LAB} = \mathrm{D1}\ \frac{\mathrm{HW\_EMP}}{\mathrm{HW\_SAL}}$$

D1 is compensation of employees in current prices. HW_SAL denotes total hours worked of employees and HW_EMP is hours worked of total employment (both based on domestic concept). Labour income for self-employed is included in gross operating surplus and mixed income (B2A3G) and cannot be easily estimated from this indicator. The formula implicitly assumes that the hourly income of self-employed equals the hourly compensation of employees. Labour and capital income shares are then calculated on the base of value added in current prices (B1G) as:

$$v_{L_t} = \frac{\mathrm{LAB}_t}{\mathrm{B1G}_t} \qquad \text{and} \qquad v_{K_t} = \frac{\mathrm{B1G}_t - \mathrm{LAB}_t}{\mathrm{B1G}_t} = 1 - v_{L_t}.$$

From these, the Divisia shares $\bar{v}_L$ and $\bar{v}_K$ are calculated. The contribution of 'crude' MFP to value added growth (also referred to as 'crude' MFP growth) is then calculated as a residual as indicated in equation (1).

At industry level, capital income share might become negative mainly due to assumptions on self-employed income, which are important in some specific industries. However, this does not happen at the total economy level and LAB can be used as a proxy for income of labour. Therefore, the 'crude' MFP growth is, so far, only provided at total economy level, including market and non-market sectors.

The MFP is also expressed as in index equal to 100 in year 2015. The index in the other years is then calculated as:

$$\mathrm{MFPIndex_t} = \mathrm{MFPIndex_{t-1}} \exp^{(\Delta \ln \mathrm{MFP_t})} \qquad \text{for } t > 2015$$

$$\mathrm{MFPIndex_t} = \mathrm{MFPIndex_{t+1}} \exp^{(-\Delta \ln \mathrm{MFP_{t-1}})} \qquad \text{for } t < 2015$$

Data sources

Member States officially send the data to Eurostat under the ESA2010 Transmission programme. B1G is gross value added (in chain linked volumes and current prices as indicated above) taken from series *nama_10_gdp (national accounts GDP and main aggregates)* in Table 1. N11N denotes the capital stock for total fixed assets (in chain linked volumes) taken from series *nama_10_nfa_st (cross-classification of fixed assets by industry and by asset (stocks))* in Table 20. Capital stock data are expressed in chain linked volumes (CLV) which are not transmitted by countries and therefore needed to be calculated from data in current and previous year replacement costs. $\mathrm{HW\_EMP}$ and HW_SAL denote the number of hours worked (in 1000) respectively by persons employed and by employees and the data are taken from series *nama_10_a10_e (employment by A*10 industry breakdowns*) in Table 1. D1 is compensation of employees in current prices taken from *nama_10_gdp (national accounts GDP and main aggregates)* in Table 1.

Availability and comparability recommendations and other information

Compared to more sophisticated approaches (e.g. OECD, EU KLEMS or some national results[10]) this 'crude' MFP, measured as a contribution to value added, is on average higher, though differences are small on average.

The advantage of the 'crude' MFP growth rate is that it is based solely on the published national accounts data and doesn't require further assumptions. Consequently, any change in quality of labour input and of capital input is part of the 'crude' MFP growth.

Appendix: Capital services and MFP growth

A more sophisticated approach when calculating MFP growth is to use 'capital services' instead of the growth rate of the (total) capital stock. Capital services growth takes into account changes in the composition and valuation of the different capital asset types. The growth rate of capital services is measured as a Törnqvist volume index of various asset types (like building, machinery, software, etc.) given by

$$\Delta \ln \mathrm{KS}_t = \sum_l \bar{v}_{\mathrm{K},l,t} \Delta \ln \mathrm{K}_{l,t}$$

where $\mathrm{K}_{l,t}$ denotes the capital stock (in chain-linked volumes) of asset type l and $\bar{v}_{\mathrm{K},l,t}$ denotes nominal (Divisia) shares in year t.

Determining these shares requires the calculation of the user-costs of capital[11], assuming a geometric depreciation profile for a cohort of assets.[12] This first requires the calculation of the *nominal rate of return* denoted by i_t using[13]

$$i_t = \frac{p_{K,t}K_t + \sum_l (p_{I,l,t} - p_{I,l,t-1})K_{l,t} - \sum_l \delta_l p_{I,l,t} K_{l,t}}{\sum_l p_{I,l,t-1} K_{l,t}}$$

where $p_K K = \mathrm{CAP}$ (i.e. capital income), and K_l is the stock of capital asset type l in chain-linked volumes. $p_{I,l,t}$ denotes the price index of gross fixed capital formation and δ_l is the depreciation rate of asset l. Ideally, these calculations are performed at the most detailed asset type classification available. The familiar *user cost-of-capital* equation[14] is then given by[15]

$$p_{K,k,t} = p_{I,k,t-1} i_t + \delta_k p_{I,k,t} - (p_{I,k,t} - p_{I,k,t-1})$$

The nominal shares are then derived as

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[10] Country-specific results often rely on more detailed data (often only available for internal use) and these more sophisticated measures are based on various specific assumptions.

[11] This is the price at which the investor is indifferent to either buying or renting the capital good for one year.

[12] This requires knowledge or assumptions of depreciation rates by asset types.

[13] Here, alternative approaches can be followed as well (see, for example, OECD, 2009, Chapter 8).

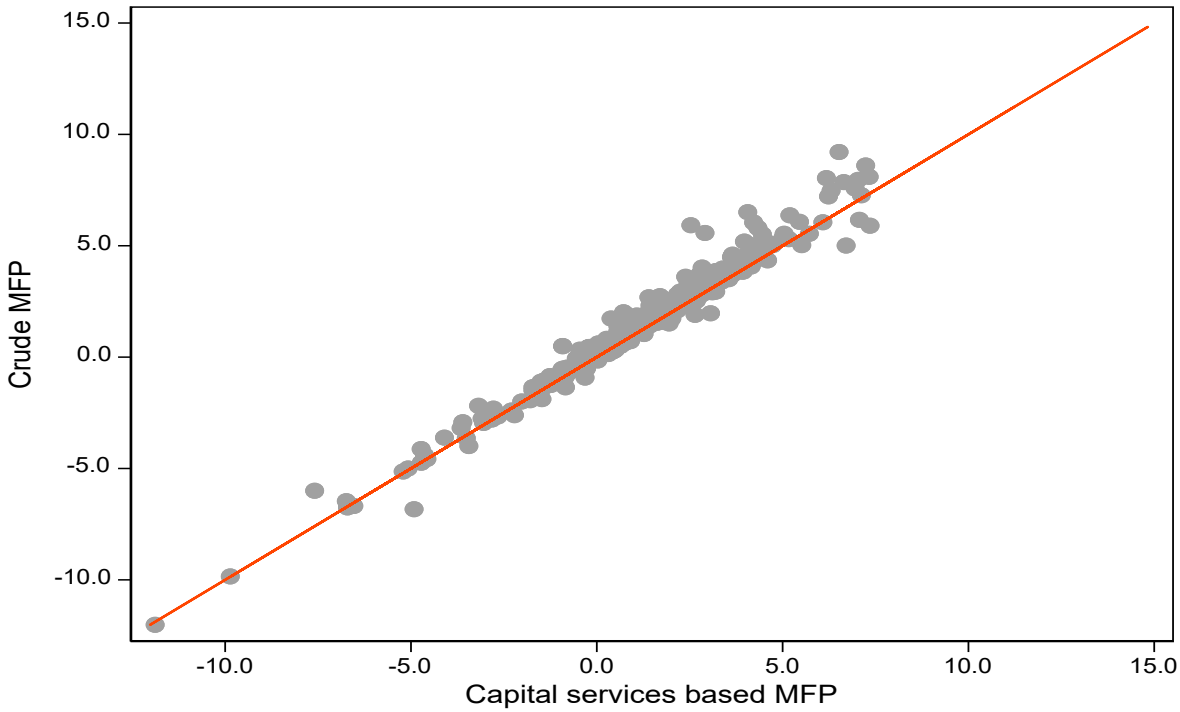
[14] For a discussion, see Jorgenson et al. (2005) or Timmer et al. (2010) for details.

[15] The capital services price can become negative, in which case it is often set to zero.

$$v_{K,k,t} = \frac{p_{K,k,t} K_{k,t}}{\sum_l p_{K,l,t} K_{k,t}}$$

It holds (by definition) that $\sum_l v_{K,l} = 1$. Using these shares, the Divisia shares $\bar{v}_{K,k,t} = (v_{K,k,t} + v_{K,k,t-1})/2$ are used to calculate capital services growth above.

The 'capital services' growth is calculated according to the method outlined above, applying the needed assumptions on the depreciation rates for the detailed asset types[16]. Normally, capital services growth is larger than growth of capital stocks, implying that the 'Crude MFP' growth is slightly larger. However, both MFP growth rates are highly correlated; this also applies for longer-term trends. The graph below shows the correlation between 'crude' MFP and capital services-based MFP growth (for yearly growth rates in EU-27 countries).



[16] For more detailed information on the assumptions and calculations, see the report on quality aspects of capital stock data drafted by The Vienna Institute for International Economic Studies (wiiw) provided in the framework of the Task Force on productivity indictors.